A new method for ranking of fuzzy numbers through using distance method

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Abstract

In this paper, by using a new approach on distance between two fuzzy numbers, we construct a new ranking system for fuzzy number which is very realistic and also matching our intuition as the crisp ranking system on R.

1. Introduction

In many fuzzy applications, defuzzification of fuzzy numbers is very important. The differences between this study and Yager [7],Yao and Wu[8] and Chen[3] are as follows: Yager[7], used a weighted mean value (or centroid, $\int_{-\infty}^{\infty} x\mu_A(x)dx \Big/ \int_{-\infty}^{\infty} \mu_A(x)dx$) to define ordering. This is different from our work and results in a difference, which is stated in section 3, example set3(see Table1).

Yao and Wu [8], used from signed distance to define ordering. It is different from our work and results in a difference, which is in section 3, example set 4 (see Table 1).

Chen [3], first normalized fuzzy numbers and then used from maximizing set and minimizing set to define ordering. It is also different from our work and results in a difference, which is stated in section 3, example set 3 (see Table 1).

First we define a fuzzy origin for fuzzy numbers then according to the distance of fuzzy numbers with respect to this origin we rank them. The basic definitions of fuzzy number are given as follows [5].

Keywords: Fuzzy numbers; Ranking of fuzzy numbers; Distance

Definition 1.1. A fuzzy number is a fuzzy set $u: R \rightarrow I=[0,1]$ which satisfies:

1. u is upper semicontinuous ,
2. $u(x)=0$ outside some interval [c,d],
3. There are real numbers a,b such that $c \leq a \leq b \leq d$ and

   3.1 $u(x)$ is monotonic increasing on $[c,a]$,

   3.2 $u(x)$ is monotonic decreasing on $[b,d]$,

   3.3 $u(x)=1, \quad a \leq x \leq b$.

The set of all the fuzzy numbers (as given by Definition 1.1) is denoted by E. An equivalent parametric is also given in [5].

Definition 1.2. A fuzzy number u in parametric form is a pair $(\underline{u},\overline{u})$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0 \leq r \leq 1$, which satisfy the following requirements:

2.1 $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,

2.2 $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,

2.3 $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$.

A popular fuzzy number is the trapezoidal fuzzy number $u(x_0, y_0, \sigma, \beta)$, with two defuzzifier x_0, y_0, and left fuzziness σ and right fuzziness β where the membership function is

$$u(x)=\begin{cases} \frac{1}{\sigma}(x-x_0+\sigma) & x_0-\sigma \leq x \leq x_0 \\ 1 & x_0 \leq x \leq y_0 \\ \frac{1}{\beta}(y_0-x+\beta) & y_0 \leq x \leq y_0+\beta \\ 0 & \textit{Otherwise.} \end{cases}$$

Definition 1.3. For arbitrary fuzzy numbers $u=(\underline{u},\overline{u})$ and $v=(\underline{v},\overline{v})$ the quantity

$$d_2(u,v)=\left[\int_0^1(\underline{u}(r)-\underline{v}(r))^2 dr+\int_0^1(\overline{u}(r)-\overline{v}(r))^2 dr\right]^{\frac{1}{2}},$$

is the distance between u and v, [1], [6].

2. Ranking of fuzzy number with distance method

Let all of fuzzy numbers be either positive or negative. Without less of generality, assume that all of them are positive. The membership function of $a \in R$ is $u_a(x) = 1$, if $x = a$; and $u_a(x) = 0$, if $x \neq a$. Hence if $a = 0$ we have the following

$$u_0(x) = \begin{cases} 1 & x = 0, \\ 0 & x \neq 0. \end{cases}$$

Since $u_0(x) \in E$, left fuzziness σ and right fuzziness β are 0, so for each $u(x) \in E$

$$d_2(u, u_0) = \left[\int_0^1 \underline{u}(r)^2 dr + \int_0^1 \overline{u}(r)^2 dr \right]^{\frac{1}{2}}.$$

Thus we have the following definition.

Definition 2.1. For u and $v \in E$, define the ranking of u and v by saying

$d(u,u_0) > d(v,u_0)$ if and only if $u \succ v$,

$d(u,u_0) < d(v,u_0)$ if and only if $u \prec v$,

$d(u,u_0) = d(v,u_0)$ if and only if $u \approx v$.

Property 2.1. Suppose u and $v \in E$ are arbitrary then:

(i) if $u = v$ then $u \approx v$,

(ii) if $v \subseteq u$ and $\underline{u}(r)^2 + \overline{u}(r)^2 > \underline{v}(r)^2 + \overline{v}(r)^2$ for all $r \in [0,1]$ then $v \prec u$.

Remark 2.1. The distance triangular fuzzy number $u = (x_0, \sigma, \beta)$ of u_0 is defined as following

$$d(u,u_0) = \left[2x_0^2 + \sigma^2/3 + \beta^2/3 + x_0(\beta - \sigma)\right]^{\frac{1}{2}}.$$

Remark 2.2. The distance trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta)$ of u_0 is defined as following

$$d(u,u_0) = \left[x_0^2 + y_0^2 + \sigma^2/3 + \beta^2/3 - x_0\sigma + y_0\beta\right]^{\frac{1}{2}}.$$

Remark 2.3. If $u \approx v$, it is not necessary that $u = v$. Since if $u \neq v$ and $(\underline{u}(r)^2 + \overline{u}(r)^2)^{\frac{1}{2}} = (\underline{v}(r)^2 + \overline{v}(r)^2)^{\frac{1}{2}}$ then $u \approx v$.

3. Discussion

A popular fuzzy number u is the symmetric triangular fuzzy number $s[x_0, \sigma]$ centered at x_0 with basis 2σ

$$u(x)=\begin{cases}\frac{1}{\sigma}(x-x_0+\sigma) & x_0-\sigma\le x\le x_0,\\ \frac{1}{\sigma}(x_0-x+\sigma) & x_0\le x\le x_0+\sigma,\\ 0 & otherwise,\end{cases}$$

which its parametric form is $\underline{u}(r)=x_0-\sigma+\sigma r$ $\quad \overline{u}(r)=x_0+\sigma-\sigma r$

Remark 3.1. The distance triangular fuzzy number $s[x_0,\sigma]$ of u_0 is defined as

$$d(s[x_0,\sigma],u_0)=[2x_0^2+\frac{2}{3}\sigma^2]^{\frac{1}{2}}.$$

If for ranking fuzzy numbers $s[x_0,\sigma]$, $s[x_0,\beta]$ and $\beta\neq\sigma$ we used Yao and Wu [8], method we would have $s[x_0,\sigma]=s[x_0,\beta]$. But with our method we have

$$s[x_0,\sigma]\neq s[x_0,\beta].$$

We shall now compare the methods used by other authors in [2], [3], [4], [7] ,[8] and our method with four sets of examples taken from Yao and Wu [8].

Set 1 : A=(0.5,0.1,0.5), B=(0.7,0.3,0.3), C=(0.9,0.5,0.1).

Set 2: A=(0.4,0.7,0.1,0.2)(trapezoidal fuzzy number) B=(0.7,0.4,0.2), C=(0.7,0.2,0.2).

Set 3: A=(0.5,0.2,0.2), B=(0.5,0.8,0.2,0.1), C=(0.5,0.2,0.4).

Set 4: A=(0.4,0.7,0.4,0.1), B=(0.5,0.3,0.4), C=(0.6,0.5,0.2).

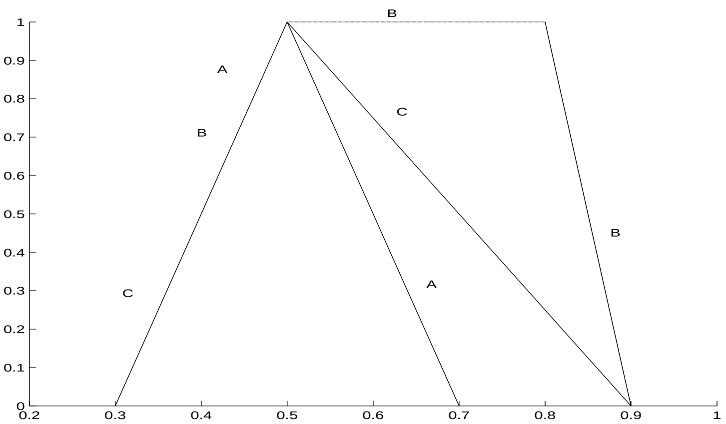

Figure 1

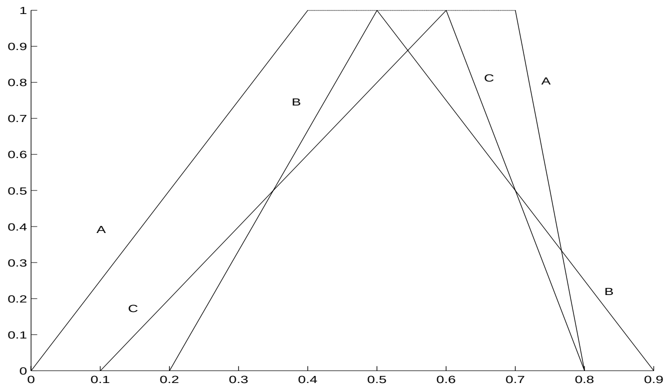

Figure 2

Table 1

set	Fuzzy number	Choobinech and Li	Yager	Chen	Baldwin and Guild	Wu
1	A B C	0.333 0.50 0.667 $A<B<C$	0.60 0.70 0.80 $A<B<C$	0.3375 0.50 0.667 $A<B<C$	0.30 0.33 0.44 $A<B<C$	0.6 0.7 0.8 $A<B<C$
2	A B C	0.458 0.583 0.667 $A<B<C$	0.575 0.65 0.7 $A<B<C$	0.4315 0.5625 0.625 $A<B<C$	0.27 0.27 0.37 $A\approx B<C$	0.575 0.65 0.7 $A<B<C$
3	A B C	0.333 0.4167 0.5417 $A<B<C$	0.5 0.55 0.625 $A<B<C$	0.375 0.425 0.55 $A<B<C$	0.27 0.37 0.45 $A<B<C$	0.5 0.625 0.55 $A<C<B$
4	A B C	0.50 0.5833 0.6111	0.45 0.525 0.55 $A<B<C$	0.52 0.57 0.625 $A<B<C$	0.40 0.42 0.42 $A<B\approx C$	0.475 0.525 0.525 $A<B\approx C$

By using our method, we have the following results:

For set 1:

$d(A,u_0)$= 0.8869, $d(B,u_0)$=1.0194, $d(C,u_0)$=1.1605, and by definition 1.2:

$A<B<C$

For set 2:

$d(A,u_0)$= 0.8756, $d(B,u_0)$=0.9522, $d(C,u_0)$=1.0033, and by Definition 1.2:

$A<B<C$

For set 3:

$d(A,u_0)$= 0.7257, $d(B,u_0)$=0.9416, $d(C,u_0)$=0.8165, and by Definition 1.2:

$A<C<B$.

For set 4:

$d(A,u_0)$= 0.7853, $d(B,u_0)$=0.7958, $d(C,u_0)$=8386, and by Definition 1.2: $A < B < C$.

4. Conclusion

In Table 1, we have the following results: In set 1, our method has the same result as in other five papers. In set 2, our method has the same result as in the other four papers. However in set 3, we and Yao and Wu have $A \prec C \prec B$, but all the other four papers have $A \prec B \prec C$. We can see from Fig.1 that define ordering $A \prec C \prec B$ is better than define ordering $A \prec B \prec C$. In set 4, Fig 2, $A \prec B \prec C$, our method leads to the same result as that of Choobinech and Li, Yager and Chen. The rest of Yao and Wu, Baldwin and Guild have $A \prec B \approx C$.

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